

## SYNTHESIS OF NONUNIFORMLY SPACED ARRAYS. USING AN ITERATIVE LINEAR METHOD.

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### Abstract :

The true problem of antenna array synthesis is, given a desired pattern, to choose the (minimal) number of elements, their placements and the weights that allow to meet it. In most cases only the last point is considered and both the number of elements and the geometry of the array is fixed. In this paper we address a simplified version of the global problem. We start with a linear array of too large a number of elements equispaced by a fraction of  $\lambda$  and optimize the weights to meet the desired pattern with the additive constraint that the maximum number of weights should be zero. We propose a quite simple iterative algorithm to solve this problem and compare our results to those in [1] where a similar problem has been considered.

### Problem formulation :

The array factor of a planar array of  $N$  elements fed with complex weights:  $I_p$ ,  $p = 1, \dots, N$ , and located at the positions:  $\vec{OM}_p = x_p \cdot \vec{u}_x + y_p \cdot \vec{u}_y$ , is given by:  $F(\theta, \varphi, I, \vec{OM}) = \sum_{p=1}^N I_p \cdot \exp\left(\frac{2i\pi}{\lambda} \cdot \vec{OM}_p \cdot \vec{s}(\theta, \varphi)\right)$  ( $\theta, \varphi$ ) are the spherical angles,  $\lambda$  is the wavelength,  $\vec{s}(\theta, \varphi)$  is the unit vector which points to the direction  $(\theta, \varphi)$ . Its modulus is proportional to the power emitted in the direction  $(\theta, \varphi)$  in the far-field. For simplicity, we restrict ourselves to real array-factors that are obtained by considering only symmetric arrays with conjugate symmetric weights. Indeed this is no limitation in case the pattern to be synthesized is itself symmetric.

The problem is then to find the symmetric array with the least number, say  $2K$ , of elements that allows to meet the desired pattern i.e. the best  $K$  complex weights and the best  $K$  element-locations.

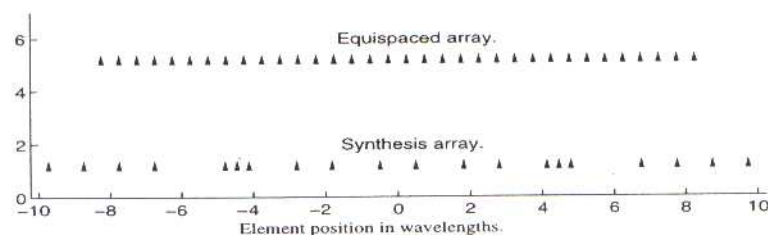
### Synthesis technique :

- We start with a linear equispaced array having  $2N$  elements and inter-element spacing  $d$ .  $2N$  is chosen much larger than necessary for the desired pattern and  $d$  smaller than  $\lambda/2$ . The design algorithm is a linear programming algorithm where the unknowns are the real and imaginary parts of the excitations  $I_p$  and the constraints guaranty the realisation of the desired pattern in  $M$  directions. We set  $K = N$ .
- iteration :  
Given the current (symmetric) array with  $2K$  elements apply the algorithm to obtain a set of weights  $\{I_p\}$  that meet the desired pattern. If no solution exists, stop and adopt the previous solution. Otherwise seek the weight with smallest modulus and discard the corresponding element (as well as the symmetrically placed element) from the array. Set  $K = K - 1$  and start the next iteration.

A number of variants can be considered to accelerate the "pruning" of the array. One can fix a threshold  $\epsilon > 0$  and discard all elements whose weights are smaller than  $\epsilon$  together with -as in the previous rule- the remaining elements with smallest "weight".

### A typical simulation result :

For comparison, we take the same example as in [1]. We first consider a linear symmetric array of 35 elements, equispaced by  $\lambda/2$ . The pattern to be synthesized can be met by this array using a Dolph-Chebyshev technique. We then apply our approach starting with an array having 116 elements equispaced by  $\lambda/3$ . Our algorithm allows to meet the same pattern using just 20 elements whose locations are shown below. In [1], a more sophisticated strategy is used to obtain a similar result.



### References :

- [1] R.M. Leahy and B.D. Jeffs, "On the Design of Maximally Sparse Beamforming Arrays," *IEEE Trans. Antennas Propagat.*, vol AP-39, pp 1178-1187, August 1991.